PMM U.S.S.R.,Vol.45,pp. 749-754 0021-8928/82/6 0749 $7.50/0

UDC 65-50

CONSTRUCTION OF A GAME'S VALUE IN CERTAIN FIXED-TIME DIFFERENTIAL GAMES*

V.I. UKHOBOTOV

A procedure is given for the construction of a maximal u-stable bridge /1/ in a fixed-time nonstationary game. A procedure is suggested for the construction of the game's value for two classes of fixed-time games. Examples are presented.

1. Let us consider a controlled process whose equations of motion are

$$z^{\cdot} = -u + v, \quad z \in R^n, \quad v \in V(t), \quad u \in U(t) \tag{1.1}$$

A segment $I = [0, p]$ of the real line is specified. For each $t \in I$ the sets $U(t)$ and $V(t)$ are compacta in R^n and are Lebesgue-measurably dependent on t on segment I /2/. A function $A(t) \geqslant 0$, summable on segment I, exists such that for each $t \in I$ the sets $U(t)$ and $V(t)$ are contained in a ball of radius $A'(t)$ with center at the origin. A closed set $X \subset R^n$ and a continuous function $g: X \to R$ whose values on set X are bounded from below by a number ε_0 are specified.

The goal of the first player, choosing the control u, is to realize the inclusion $z(p) \in X$ and to make the value of $g(z(p))$ as small as possible. The second player's goal is the opposite. The game is played with discrimination of the second player /3,4/. The method proposed in /3/ for stationary games will be used to find the game's value.

Let a set $Z \subset R^n$ and the numbers $0 \leqslant t \leqslant \tau \leqslant p$ be prescribed. We denote

$$T_t^{\tau}(Z) = \Big(Z + \int_t^{\tau} U(r)\,dr\Big) \overset{*}{-} \int_t^{\tau} V(r)\,dr \tag{1.2}$$

Here $\overset{*}{-}$ denotes the geometric difference of two sets /4/. For each $\varepsilon \geqslant \varepsilon_0$ we consider the set

$$X(\varepsilon) = \{z \in X : g(z) \leqslant \varepsilon\} \tag{1.3}$$

We construct a maximal u-stable bridge /1/ $W(t, \varepsilon)$ going onto set (1.3) at the instant p. This means that: 1) $W(p, \varepsilon) = X(\varepsilon)$; 2) $T_t^{\tau}(W(\tau, \varepsilon)) \supset W(t, \varepsilon)$ for all $0 \leqslant t \leqslant \tau \leqslant p$; 3) if the point $z(0) \overline{\in} W(0, \varepsilon)$, then a finite collection of numbers $0 < \tau_1 < \ldots < \tau_m = p$ exists such that

$$z(0) \overline{\in} T_0^{\tau_1}(T_{\tau_1}^{\tau_2}(\ldots T_{\tau_m}^{p}(X(\varepsilon))\ldots))$$

For each $0 \leqslant t \leqslant p$ we set

$$W^1(t, \varepsilon) = T_t^p(X(\varepsilon)), \ldots, W^{k+1}(t, \varepsilon) = \bigcap_{t \leqslant \tau \leqslant p} T_t^{\tau}(W^k(\tau, \varepsilon)) \tag{1.4}$$

As was done in /5/ for stationary games, we can show that

$$W(t, \varepsilon) = \bigcap_{k \geqslant 1} W^k(t, \varepsilon) \tag{1.5}$$

Lemma 1.1. Let numbers $0 \leqslant t(\varepsilon) < p$ and $k \geqslant 1$ exist such that $W^k(t, \varepsilon) = W^{k+1}(t, \varepsilon)$ for $t(\varepsilon) \leqslant t \leqslant p$. Then $W(t, \varepsilon) = W^k(t, \varepsilon)$ for $t(\varepsilon) \leqslant t \leqslant p$.

Lemma 1.2. Let the hypotheses of the preceding lemma be fulfilled with $t(\varepsilon) > 0$ and let a sequence $t_i \to t(\varepsilon)$, $t_i < t(\varepsilon)$, exist such that the sets $W^k(t_i, \varepsilon)$ are empty. Then $W(t, \varepsilon) = W^k(t, \varepsilon)$ for $t(\varepsilon) \leqslant t \leqslant p$ and the sets $W(t, \varepsilon)$ are empty for $0 \leqslant t \leqslant t(\varepsilon)$.

These lemmas are proved with the use of equalities (1.4) and (1.5) /5/.

Let the initial value $z(0) = z$ be prescribed. Then /3/ the value $G(z)$ of the game being analyzed equals

$$G(z) = \inf\{\varepsilon \geqslant \varepsilon_0 : z \in W(0, \varepsilon)\} \tag{1.6}$$

The general arguments presented will be used below to find the game's value in concrete classes

*Prikl.Matem.Mekhan.,45,No.6,994-1000,1981 of games. We note that another method, based on sequential procedures of game value construction, was examined, for example, in /6 – 8/.

2. Let us consider the case when the constraints imposed on the choice of control u and the payoff $g: R^n \to R$ have a special form, and the terminal set is $X = R^n$. The vectors $x_1, \ldots x_n, x_{n+1}$ from R^n are specified and the first n of them are linearly independent, while the coefficients f_i in the expansion $x_{n+1} = f_1x_1 + \ldots + f_nx_n$ are negative. Nonnegative and continuous scalar functions $a_i(t)$ $(i = 1, \ldots, n+1)$ are prescribed on segment $[0, p]$. Then, denoting the scalar product in R^n by (x, u), we take

$$U(t) = \{u \in R^n : (x_i, u) \leqslant a_i(t),\ i = 1, \ldots, n+1\} \tag{2.1}$$

$$g(z) = \max_{1 \leqslant i \leqslant n+1} (x_i, u) \tag{2.2}$$

From the constraints imposed on vectors x_i it follows that $g(z) \geqslant 0$ for any $z \in R^n$ and ($|\cdot|$ is the vector's length)

$$g(z) > 0 \text{ for } |z| > 0 \tag{2.3}$$

We note certain properties of polyhedrons of form (2.1). Let the numbers $b_1^k, \ldots, b_{n+1}^k$ $(k = 1, 2, 3)$ be prescribed. We set

$$B^k = \{z \in R^n : (x_i, z) \leqslant b_i^k,\ i = 1, \ldots, n+1\}\ (k = 1, 2, 3) \tag{2.4}$$

Lemma 2.1. Polyhedron (2.4) is not empty then and only then

$$b_{n+1}^k - \sum_{i=1}^n f_i b_i^k \geqslant 0 \tag{2.5}$$

Proof. Polyhedron (2.4) is not empty then and only then /9/ there do not exist $\lambda_i \geqslant 0$ such that

$$\sum_{i=1}^{n+1} \lambda_i x_i = 0, \quad \sum_{i=1}^{n+1} \lambda_i b_i^k = -1$$

Substituting $x_{n+1} = f_1x_1 + \ldots + f_nx_n$ into the first equality and taking into account that the coefficients $f_i < 0$ and that the vectors x_i $(i = 1, \ldots, n)$ are linearly independent, we obtain the lemma's assertion.

Lemma 2.2. Let inequality (2.5) be fulfilled, then polyhedron (2.4) is bounded.

Proof. From inequality (2.3) it follows that $\min g(z) = m > 0$, where the minimum is taken over all $z \in R^n, |z| = 1$. For an arbitrary vector $z \in B^k$ we have $|z| m \leqslant g(z) \leqslant \max(b_i^k)$.

Lemma 2.3. Let three not empty polyhedron (2.4) be prescribed, and let $b_i^3 = b_i^1 + b_i^2$ for $i = 1, \ldots, n+1$. Then $B^3 = B^1 + B^2$.

Proof. The inclusion $B^1 + B^2 \subset B^3$ follows from the definition of the sum of sets. Let the point $z \in B^3$. To prove the inclusion $z \in B^1 + B^2$ it is sufficient to find a point $x \in B^1$ such that $z - x \in B^2$. Such a point $x \in B^1$ exists if the following system of inequalities is consistent:

$$(x_i, x) \leqslant b_i^1,\ (-x_i, x) \leqslant b_i^2 - (x_i, z),\ i = 1, \ldots, n+1 \tag{2.6}$$

For system (2.6) to be consistent it is necessary and sufficient /9/ that there do not exist $\lambda_i \geqslant 0, \mu_i \geqslant 0$ such that

$$\sum_{i=1}^{n+1} (\lambda_i - \mu_i) x_i = 0, \quad \sum_{i=1}^{n+1} (\lambda_i b_i^1 + \mu_i b_i^2 - \mu_i (x_i, z)) = -1 \tag{2.7}$$

We take $\lambda_i \geqslant 0, \mu_i \geqslant 0$ satisfying the first equality in (2.7) and we show that

$$\sum_{i=1}^{n+1} \mu_i (x_i, z) \leqslant \sum_{i=1}^{n+1} (\lambda_i b_i^1 + \mu_i b_i^2) \tag{2.8}$$

This proves that the second equality in (2.7) is not fulfilled. By hypothesis the point z satisfies the system of inequalities $(x_i, z) \leqslant b_i^1 + b_i^2$, $i = 1, \ldots, n+1$. Multiplying these inequalities by λ_i, μ_i, summing, and taking into account the first equality in (2.7), we get that inequality (2.8) is fulfilled if the right hand side in (2.8) is not less than even one of the following two numbers:

$$\sum_{i=1}^{n+1} \mu_i (b_i^1 + b_i^2), \quad \sum_{i=1}^{n+1} \lambda_i (b_i^1 + b_i^2)$$

It remains to show that at least one of the following inequalities is valid:

$$\sum_{i=1}^{n+1} (\lambda_i - \mu_i)\, b_i^1 \geqslant 0, \quad \sum_{i=1}^{n+1} (\mu_i - \lambda_i)\, b_i^2 \geqslant 0 \tag{2.9}$$

From the first equality in (2.7), allowing for the expansion of vector x_{n+1}, we obtain $\lambda_i - \mu_i = (\mu_{n+1} - \lambda_{n+1}) f_i$. We rewrite inequality (2.9) in the equivalent form

$$(\mu_{n+1} - \lambda_{n+1})\Big(\sum_{i=1}^{n} f_i b_i^1 - b_{n+1}^1\Big) \geqslant 0, \quad (\lambda_{n+1} - \mu_{n+1})\Big(\sum_{i=1}^{n} f_i b_i^2 - b_{n+1}^2\Big) \geqslant 0$$

From here and from the condition not empty (2.5) it follows that one of the inequalities in (2.9) is valid.

From the lemmas we have proved it follows that for each $t \in [0, p]$ the set (2.1) is a not empty convex compactum in R^n. Using Lemma 2.3 we can show that

$$\int_{t_1}^{t_2} U(t)\,dt = \Big\{z \in R^n : (x_i, z) \leqslant \int_{t_1}^{t_2} a_i(t)\,dt,\ i = 1, \ldots, n+1\Big\} \tag{2.10}$$

for any $0 \leqslant t_1 \leqslant t_2 \leqslant p$.

Lemma 2.4. Let B be a compactum in R^n, while $b_i = \max\,(x_i, z)$, where the maximum is taken over $z \in B$. Then $B^1 \overset{*}{-} B = B^2$ with the numbers $b_i^2 = b_i^1 - b_i$.

The proof is analogous to that of Lemma 2.1 in /5/.

Let us now to construct the sets (1.2) and (1.5) in the game at hand. From formula (2.2) it follows that for each $\varepsilon \geqslant 0$ the set (1.3) has the form

$$X(\varepsilon) = \{z \in R^n\colon\ (x_i, z) \leqslant \varepsilon,\ i = 1, \ldots, n+1\} \tag{2.11}$$

We denote

$$b_i(t) = \max_v (x_i, v)\ (v \in V(t)), \quad \nu_i(t) = \int_t^p (a_i(\tau) - b_i(\tau))\,d\tau \tag{2.12}$$

Then, using the two preceding lemmas and equalities (2.10), (2.11), from formulas (1.2), (1.4) we obtain

$$W^1(t, \varepsilon) = \{z \in R^n : (x_i, z) \leqslant \varepsilon + \nu_i(t),\ i = 1, \ldots, n+1\} \tag{2.13}$$

We denote

$$t(\varepsilon) = \inf\Big\{t \geqslant 0 : \varepsilon + \nu_{n+1}(\tau) \geqslant \sum_{i=1}^{n} (\varepsilon + \nu_i(\tau)) f_i \ \text{ for } \ t \leqslant \tau \leqslant p\Big\} \tag{2.14}$$

Then, as follows from Lemma 2.1, the set (2.13) is not empty for all $t(\varepsilon) \leqslant t \leqslant p$. Using the same arguments as in the proof of equality (2.13), we can show that $T_t^p(X(\varepsilon)) = T_t^\tau(T_\tau^p(X(\varepsilon)))$ for $t(\varepsilon) \leqslant t \leqslant \tau \leqslant p$. Hence from (1.4) it follows that $W^2(t, \varepsilon) = W^1(t, \varepsilon)$ for $t(\varepsilon) \leqslant t \leqslant p$. Let $t(\varepsilon) = 0$. Then from Lemma 1.1 we obtain that $W(t, \varepsilon) = W^1(t, \varepsilon)$ for $0 \leqslant t \leqslant p$. If $t(\varepsilon) > 0$, then from the definition (2.14) of the number $t(\varepsilon)$, equality (2.13) and Lemma 2.1 follows the existence of the sequence of numbers $t_i \to t(\varepsilon), t_i < t(\varepsilon)$, such that the sets $W^1(t_i, \varepsilon)$ are empty. By Lemma 1.2 the set $W(t, \varepsilon)$ is empty for $0 \leqslant t \leqslant t(\varepsilon)$, while for $t(\varepsilon) \leqslant t \leqslant p$ it coincides with set (2.13).

The value $G(z)$ from (1.6) of the game being examined equals the smallest of the numbers $\varepsilon \geqslant 0$ satisfying the following two conditions:

$$t(\varepsilon) = 0, \quad \max_{1 \leqslant i \leqslant n+1} ((x_i, z) - \nu_i(0)) \leqslant \varepsilon \tag{2.15}$$

The first one of them signifies that $W(0, \varepsilon) = W^1(0, \varepsilon)$. As follows from (2.13), the second condition signifies the inclusion $z \in W^1(0, \varepsilon)$.

Example 2.1. The equations of motion describing the game are

$$x^{\cdot\cdot} = w,\ x \in R^n,\ (x_i, w) \leqslant \delta_i,\ i = 1, \ldots, n+1$$
$$y^{\cdot} = v,\ y \in R^n,\ |v| \leqslant \lambda$$

An instant $p > 0$ is specified. The first player, choosing control w, strives to minimize the quantity $\max[(x_i, y(p) - x(p))]$, where the maximum is taken over $i = 1, \ldots, n+1$. We introduce new variables: $z = y - x - (p - t)x^{\cdot}$, $u = (p - t)w$. We obtain the equivalent game

$$g(z) = \max(x_i, z),\ z^{\cdot} = -u + v,\ (x_i, u) \leqslant (p - t)\delta_i,\ |v| \leqslant \lambda$$

In the present example the functions (2.12) equal

$$b_i(t) = \lambda |x_i| = b_i,\ \nu_i(t) = 2^{-1}(p - t)^2\delta_i - (p - t)b_i$$

We denote

$$\delta = \Big(\delta_{n+1} - \sum_{i=1}^n f_i\delta_i\Big)\Big(1 - \sum_{i=1}^n f_i\Big)^{-1}, \quad b = \Big(b_{n+1} - \sum_{i=1}^n f_i b_i\Big)\Big(1 - \sum_{i=1}^n f_i\Big)^{-1}$$

Then the conditions (2.15) for finding the game's value become

$$\varepsilon \geqslant bt - (\delta t^2)/2,\ 0 \leqslant t \leqslant p$$
$$\varepsilon \geqslant \max_{1 \leqslant i \leqslant n+1}((x_i, z) - (\delta_i p^2)/2 + b_i) = \Phi(z, p)$$

Hence we determine the game's value

$$G(z) = \max(\Phi(z, p);\ bp - (\delta p^2)/2),\ p \leqslant b/\delta$$
$$G(z) = \max(\Phi(z, p);\ b^2/(2\delta)),\ p > b/\delta$$

3. Let us consider the case when $V(t) = \alpha(t)S$ and

$$U(t) = \{u \in R^n: a_i(t) \leqslant (x_i, u) \leqslant A_i(t),\ i = 1, \ldots, n\} \tag{3.1}$$

Here $x_1, \ldots, x_n$ is some basis in R^n, $a_i(t) \leqslant A_i(t)$ $(i = 1, \ldots, n)$ are continuous functions on $[0, p]$, S is a convex compactum in R^n, containing the origin as an interior point, and $\alpha(t)$ is a continuous and nonnegative scalar function defined on $[0, p]$. The game's payoff is specified with the aid of the Minkowski function $g(z)$ of set S:

$$g(z) = \inf\{\varepsilon > 0:\ z \in \varepsilon S\} \Rightarrow X(\varepsilon) = \{z: g(z) \leqslant \varepsilon\} = \varepsilon S \tag{3.2}$$

Lemma 3.1. Let B be a closed convex set in R^n and let ε and δ be nonnegative numbers. Then

$$(\varepsilon S + B) \stackrel{*}{-} \delta S = \begin{cases} (\varepsilon - \delta)S + B, & \varepsilon \geqslant \delta \\ B \stackrel{*}{-} (\delta - \varepsilon)S, & \varepsilon < \delta \end{cases}$$

The lemma can be proved by bringing in the notion of the support function of a closed convex set.

We denote

$$p(\varepsilon) = \inf\Big\{t: 0 \leqslant t \leqslant p,\ \varepsilon \geqslant \int_t^p \alpha(\tau)\,d\tau\Big\} \tag{3.3}$$

Then, as follows from Lemma 3.1 and formulas (1.2), (1.4),

$$W^1(t, \varepsilon) = \Big(\varepsilon - \int_t^p \alpha(\tau)\,d\tau\Big)S + \int_t^p U(\tau)\,d\tau, \quad p(\varepsilon) \leqslant t \leqslant p \tag{3.4}$$

$$W^1(t, \varepsilon) = \Big(\int_t^p U(\tau)\,d\tau\Big) \stackrel{*}{-} \int_t^{p(\varepsilon)} \alpha(\tau)\,d\tau S, \quad t \leqslant p(\varepsilon) \tag{3.5}$$

We denote $B_i = \max(x_i, s)$, $b_i = \min(x_i, s)$ (the maximum and minimum are taken over $s \in S$) and for $0 \leqslant t \leqslant p(\varepsilon)$:

$$\nu_i(t, \varepsilon) = \int_t^p a_i(\tau)\,d\tau - b_i \int_t^{p(\varepsilon)} \alpha(\tau)\,d\tau, \tag{3.6}$$

$$\mu_i(t, \varepsilon) = \int_t^p A_i(\tau)\,d\tau - B_i \int_t^{p(\varepsilon)} \alpha(\tau)\,d\tau$$

Then, as was shown in /5/, a set of form (3.5) is specified by the inequalities

$$W^1(t,\varepsilon)=\{z\in R^n: \nu_i(t,\varepsilon)\leqslant(x_i,z)\leqslant\mu_i(t,\varepsilon),\ i=1,\ldots,n\} \tag{3.7}$$

We set

$$t(\varepsilon)=\inf\{t: 0\leqslant t\leqslant p(\varepsilon),\ \nu_i(\tau,\varepsilon)\leqslant\mu_i(\tau,\varepsilon)\quad\text{for}\quad t\leqslant\tau\leqslant p(\varepsilon),\ i=1,\ldots,n\} \tag{3.8}$$

Then for $t(\varepsilon)\leqslant t\leqslant p$ the sets $W^1(t,\varepsilon)$, and for $t(\varepsilon)\leqslant t\leqslant p(\varepsilon)$ they have the form (3.7), while for $p(\varepsilon)\leqslant t\leqslant p$, the form (3.4). Using Lemma 3.1, as well as Lemmas 2.1 and 2.2 from /5/, we can prove the equality $W^2(t,\varepsilon)=W^1(t,\varepsilon)$ for $t(\varepsilon)\leqslant t\leqslant p$ and, therefore, $W(t,\varepsilon)=W^1(t,\varepsilon)$. If $t(\varepsilon)>0$, then, as in Sect.2, we get that the sets $W(t,\varepsilon)$ are empty for $0\leqslant t\leqslant t(\varepsilon)$. The game's value is found from condition (1.6).

Example 3.1. Consider Example 2.1. We shall reckon that the constraints on v are the same, but the constraints on w and on the payoff $g(z)$ have the form

$$-\delta\leqslant w_i\leqslant\delta\ (i=1,\ldots,n),\quad g(z)=|z|$$

If we denote $x_i=(0,\ldots,1,\ldots,0)$ (the one is at the i-th place), as S we take a Euclidean ball of unit radius, and pass, as in Example 2.1, to the variable z, then we obtain the equivalent game

$$z^{\cdot}=-u+v,\quad -(p-t)\delta\leqslant(x_i,u)\leqslant(p-t)\delta,\quad v\in\lambda S \tag{3.9}$$

The number $p(\varepsilon)=p-\varepsilon/\lambda$ for $0\leqslant\varepsilon\leqslant\lambda p$ and $p(\varepsilon)=0$ for $\varepsilon\geqslant\lambda p$. Therefore, for $\varepsilon\geqslant\lambda p$ the set $W(0,\varepsilon)$ is specified by the right-hand side of equality (3.4) with $t=0$. If we denote $Q=\{z\in R^n: -1\leqslant(x_i,z)\leqslant 1,\ i=1,\ldots,n\}$, then, as follows from (3.9),

$$U(t)=(p-t)\delta Q,\quad \int_t^p U(\tau)\,d\tau=2^{-1}\delta p^2Q \tag{3.10}$$

Consequently

$$W(0,\varepsilon)=(\varepsilon-\lambda p)S+2^{-1}\delta p^2Q,\quad \varepsilon\geqslant\lambda p \tag{3.11}$$

Let $\varepsilon<\lambda p$. Then functions (3.6) become

$$\mu_i(t,\varepsilon)=-\nu_i(t,\varepsilon)=2^{-1}\delta(p-t)^2-\lambda(p-t)+\varepsilon,\quad 0\leqslant t\leqslant p-\varepsilon/\lambda \tag{3.12}$$

From (3.8) and (3.12) it follows that the condition for set $W(0,\varepsilon)$ to be nonempty or, what is the same, the condition $t(\varepsilon)=0$ takes the form

$$2^{-1}\delta\tau^2-\lambda\tau+\varepsilon\geqslant 0,\quad \varepsilon/\lambda\leqslant\tau\leqslant p\quad(\varepsilon<\lambda p) \tag{3.13}$$

Having studied inequality (3.13), the nonemptiness condition for set $W(0,\varepsilon)$ when $\varepsilon<\lambda p$ can be written as

$$\begin{aligned}&-2^{-1}\delta p^2+\lambda p\leqslant\varepsilon, && p\leqslant\lambda/\delta\\ &\lambda^2/(2\delta)\leqslant\varepsilon, && p>\lambda/\delta\end{aligned} \tag{3.14}$$

Under these conditions, setting $t=0$ in (3.7) and allowing for (3.12) and the equality $W(0,\varepsilon)=W^1(0,\varepsilon)$, we obtain

$$W(0,\varepsilon)=(2^{-1}\delta p^2-\lambda p+\varepsilon)Q \tag{3.15}$$

We denote

$$\begin{aligned}&\varphi(\varepsilon,p)=\varepsilon-\lambda p,\quad f(\varepsilon,p)=2^{-1}\delta p^2,\quad \varepsilon\geqslant\lambda p\\ &\varphi(\varepsilon,p)=0,\quad f(\varepsilon,p)=2^{-1}\delta p^2-\lambda p+\varepsilon,\quad \varepsilon<\lambda p\end{aligned} \tag{3.16}$$

Set $W(0,\varepsilon)$ is nonempty then and only then inequalities (3.14) are fulfilled. In this case, as follows from (3.11), (3.15) and (3.16)

$$W(0,\varepsilon)=\varphi(\varepsilon,p)S+f(\varepsilon,p)Q \tag{3.17}$$

The support function of set Q equals $|\psi_1|+\ldots+|\psi_n|$, where $\psi=(\psi_1,\ldots,\psi_n)$. Therefore, point z belongs to set (3.17) then and only then /9/

$$\max\Big((z,\psi)-f(\varepsilon,p)\sum_{i=1}^n|\psi_i|\Big)\leqslant\varphi(\varepsilon,p) \tag{3.18}$$

Here the maximum is taken over all vectors $\psi=(\psi_1,\ldots,\psi_n)$ of unit Euclidean length. Therefore, the game's value $G(z)$ for the initial position z is determined as the smallest of numbers $\varepsilon\geqslant 0$ satisfying inequalities (3.14) and (3.18).

REFERENCES

1. KRASOVSKII N.N., and SUBBOTIN A.I., Positional Differential Games. Moscow, NAUKA, 1974.
2. HERMES H., The Generalized Differential Equation $x^{\cdot} \in R\ (t,\ x)$. Adv. Math., Vol.4, No.2,1970.
3. PSHENICHNYI B.N. and SAGAIDAK M.I., On fixed-time differential games. KIBERNETIKA, No.2, 1970.
4. PONTRIAGIN L.S., On linear differential games. 2. Dokl. Akad. Nauk SSSR, Vol.175, No.4, 1967.
5. UKHOBOTOV V.I., On the construction of stable bridges. PMM Vol.44, No.5, 1980.
6. CHENTSOV A.G., On the structure of an encounter game problem. Dokl. Akad. Nauk SSSR, Vol. 224, No.6, 1975.
7. CHENTSOV A.G., On a game problem of encounter at a prescribed instant. Mat. Sb., Vol.99, No.3, 1976.
8. CHISTIAKOV S.V., On solving pursuit game problems. PMM Vol.41, No.5, 1977.
9. PSHENICHNYI B.N., Convex Analysis and Extremal Problems. Moscow, NAUKA, 1980.

Translated by N.H.C.